(UNIT-02) : (Lecture-01)

Regular Expression & Language

Regular Expression is used to define a regular language into algebraic discription.

The operators of Regular Expression

There are three operator which are used to represent Regular Expression

1. **Union**: Union of two language L_1 & L_2 denoted by $L_1 \cup L_2$, is the set of string that are in either L_1 or L_2, or both

Exp $L_1 = \{001, 10, 111\}$ & $L_2 \{\epsilon, 001\}$

$L_1 \cup L_2 = \{\epsilon, 001, 10, 111\}$

2. ~~Contents~~

2. **Concatenation**: The Concatenation of languag L_1 & L_2 is represented by $L_1 \cdot L_2$. the Concatenation of two language represented by product of the language.

$L_1 = \{001, 10, 111\}$ & $L_2 \{\epsilon, 001\}$

$L_1 \cdot L_2 = \{001, 10, 111, 001001, 10001, 111001\}$

3. **Closure** (star, Kleene closure): The Closure of a language L is denoted by L^* and represents the set of those string that can be formed by taking any no. of string from L, possible with repetitions & Concatenating all of them

Exp. $L = \{0, 1\}$, the L^* is all string of 0's & 1's including Null string

4. The Positive closure of a language L is denoted by L^+ & represent the set of those string that can be formed by taking any no. of string from L, possible with repetitions & Concatenating all of them, excluding null string.

$$L^+ = L^* - \epsilon$$

Definition of Regular Expression:

The set of regular expression is define by the following rules:

1. Every latter of Σ Can be made into a regular expression, Null string, ϵ itself is a regular expression.

2. if r_1 & r_2 are regular expression then.

(i) (r_1) (ii) $r_1 \cdot r_2$ (iii) $r_1 + r_2$ (iv) r_1^* (v) r_2^* (vi) r_1^+ are also regular expression.

3. ~~Nothin~~ Nothing else is regular expression.

Q.1. Write Regular expression over alphabate (a, b, c) Containing at least one a and at least one b.

Soln.

$$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* + (a+b+c)^* b (a+b+c)^* a (a+b+c)^*$$

Q.2. Write the regular expression for the set of string of 0's & 1's Whose tenth symbole from the right end is 1.

Ans :-

r :- $(0+1)^* 1 (0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$

$r \rightarrow (0+1)^* 1 (0+1)^9$

Q.3 Write the regular expression for the set of strings of an equal no. of 0's & 1's such that in every prefix, the no. of 0's differs from the the no. of 1's by at most 1.

Ans.

$r = (01 + 10)^*$

Q.4. Write a regular expression for the set of string of 0's and 1's not Containing 101 as a substring.

Ans. We can analyse the set of strings then it becomes Clear that string may start with 0 & 0's may be repeated, Wherever one is encountered then no sigle 0 must follow it so string are like 0000010, 0000 111111, 100100l

$RE \rightarrow (0^* 1 00)^* 0^* 1$

Q5- Write the regular expression over alphabate {a,b} for the set of String with even no. of a's followed by odd no. of b's

Sol.

$$(aa)^* (bb)^* b$$

$$L = a^{2n} b^{2m+1} \mid n, m \geq 0$$

Q6- Write the regular Expression for language even length of string on alphabate $\Sigma = \{a,b\}$

RE → $((a+b)(a+b))^*$

or

for odd length

RE → $((a+b)(a+b))^* (a+b)$

Q7 Find the RE for the following Language.

(i) $L = \{$ length of all string divisible by 3 for $\Sigma = \{a,b\}\}$

RE → $((a+b)(a+b)(a+b))^*$

(ii) $L = \{$ length of all string $\cong 2 \bmod 3$ for $\Sigma = \{a,b\}\}$

RE → $((a+b)(a+b)(a+b))^* (a+b)(a+b)$

(iii) $L = \{$ length of string exactly 2's a for $\Sigma \{a,b\}\}$

RE → $b^* a\, b^* a\, b^*$

(iv) $L = \{$ a are at lest 2 for $\Sigma \{a, b\}\}$

$$(a+b)^* a (a+b)^* a (a+b)^*$$

(v) $L = \{$ a are at most 2 for $\Sigma = \{a, b\}\}$

RE → $b^* (\epsilon + a)\ b^*\ (\epsilon + a)\ b^*$

(Q) Write RE over alphabate $\Sigma \{a, b\}$ for the language.

$L = \{$ string starting & ending with same symbole $\}$

RE → $a (a+b)^* a + b (a+b)^* b$

Arden's Theorem:

Let P and Q be two regular expression over alphabate Σ. if P does not contain null string ϵ, then

$$R = Q + RP$$

has a unique solution that is $R = QP^*$

it can be understand as: $R = Q + RP$

Put the value of R in R.H.S.

$$R = Q + (Q + RP)P$$

When we put the value again & again

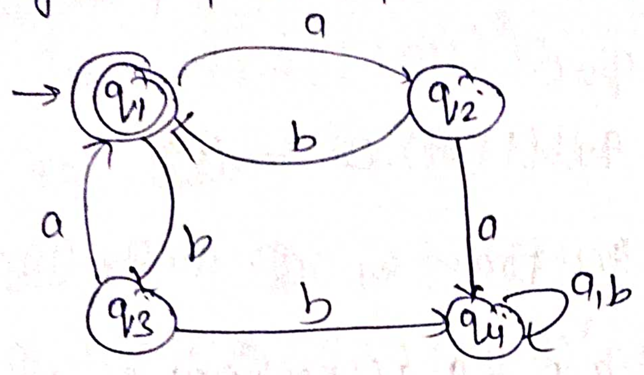
$$= Q + QP + (Q + RP)P^2$$

$$= Q + QP + QP^2 + QP^3 + \cdots$$

$$= Q(\epsilon + P + P^2 + P^3 + \cdots)$$

$$\boxed{R = QP^*}$$

Q. Find the regular expression for transition diagram given.



Ans.

$$q_1 = q_2 b + q_3 a + \epsilon \quad \text{--- ①}$$

$$q_2 = q_1 a \quad \text{--- ②}$$

$$q_3 = q_1 b \quad \text{--- ③}$$

$$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \quad \text{--- ④}$$

Keep the value of Eq. ③ & Eq. ② into Eq. ①

$$q_1 = q_1 ab + q_1 aa + \epsilon$$

$$q_1 = q_1 (ab + ba) + \epsilon$$

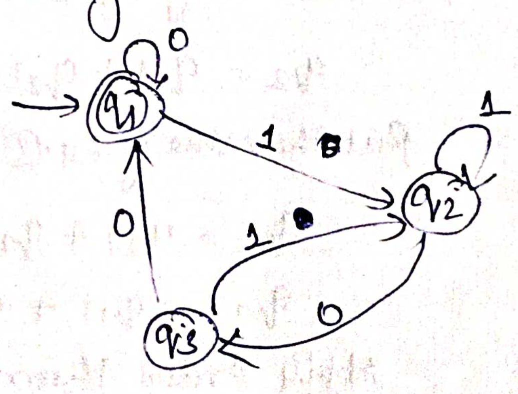
$$q_1 = \underline{\epsilon} + \underline{q_1}\underline{(ab+ba)}$$

$$R = Q + R\,P$$

$$q_1 = \epsilon (ab+ba)^* = (ab+ba)^*$$

$$RE \Rightarrow (ab+ba)^*$$

Q. Construct a RE corresponding to the state diagram.



Ans:

$$q_1 = q_1 0 + q_3 0 + \epsilon \quad \text{--- ①}$$

$$q_2 = q_1 1 + q_2 1 + q_3 1 \quad \text{--- ②}$$

$$q_3 = q_2 0 \quad \text{--- ③}$$

~~Keep the~~ Put the value of q_3 in Eq. ②

$\cancel{q_1 = q_1 0 + q_2 1 0}$

$\cancel{q_1 = q_1 0} +$

$$q_2 = q_1 1 + q_2 1 + q_2 01$$

$$q_2 = q_1 1 + q_2 (1 + 01)$$

$$q_2 = q_1 1 (1+01)^* \quad \text{--- ④}$$

Keep the value of q_3 from Eq. ④ to Eq. ③

$$q_3 = q_2 0$$
$$q_3 = q_1 1(1+01)^* 0 \quad \text{—— ⑤}$$

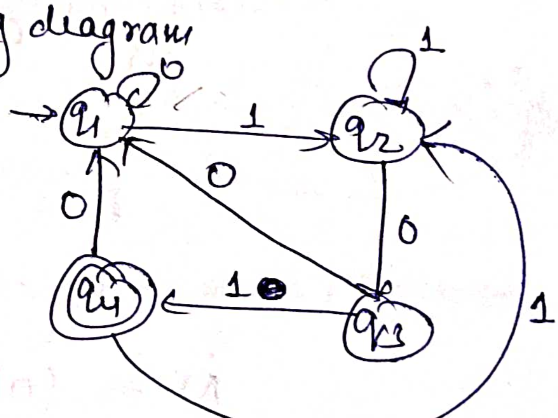
Keep the value of q_3 from Eq. ⑤ to Eq. ①

$$q_1 = q_1 0 + q_1 1(1+01)^* 00 + \epsilon$$
$$q_1 = q_1(0+1(1+01)^* 00) + \epsilon$$
$$q_1 = \epsilon + q_1(0+1(1+01)^* 00)$$

Apply the arden's theorem

$$R = Q + RP \Rightarrow QP^*$$
$$q_1 = (0+1(1+01)^* 00)^*$$

Q. Find the regular expression corresponding diagram



$$q_1 = \epsilon + q_1 0 + q_3 0 + q_4 0 \quad \text{—— ①}$$
$$q_2 = q_1 1 + q_2 1 + q_4 \cdot 1 \quad \text{—— ②}$$
$$q_3 = q_2 0 \quad \text{—— ③}$$
$$q_4 = q_3 1 \quad \text{—— ④}$$

Put the value of Eq. ④ into Eq. 2.

$$q_2 = q_1 1 + q_2 1 + q_3 11 \quad \text{—— ⑤}$$

Put the value of Eq. ③ in Eq. ⑤

$$q_2 = q_1 1 + q_2 1 + q_2 011$$
$$q_2 = q_1 1 + q_2(1+011)$$

Apply Arden theorem -

$$q_2 = q_1 1(1+011)^* \quad \text{—— ⑥}$$

Put the value of Eq. ⑥ into Eq. ③

$$q_3 = q_1 1(1+011)^* 0 \quad \text{—— ⑦}$$

Put the value of Eq. 7 into Eq. 4

$$q_4 = q_1 1(1+011)^* 0 \quad \text{--- (8)}$$

Put the eq. 8 and eq. 7 value into Eq. 1

$$q_1 = \epsilon + q_1 0 + q_1 1(1+011)^* 00 + q_1 1(1+011)^* 00$$

$$q_1 = \epsilon + q_1 (0 + 1(1+011)^* 00 + 1(1+011)^* 00)$$

Apply Arden theorem

$$q_1 = (0 + 1(1+011)^* 00 + 1(1+011)^* 00)^* \quad \text{--- (9)}$$

Put the value of Eq. 9 into Eq. 8

$$\cancel{q_1 = q_1}$$

$$q_4 = (0 + 1(1+011)^* 00 + 1(1+011)^* 00)^* 1(1+011)^* 0$$

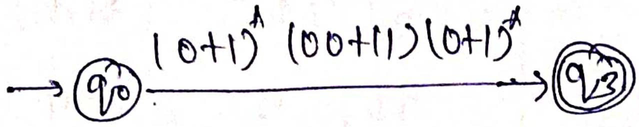
RE $\Rightarrow$ $(0+1(1+011)^* 00 + 1(1+011)^* 00)^* 1(1+011)^* 0$

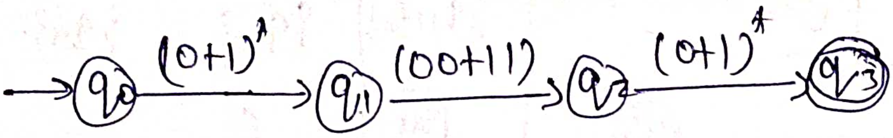
(Lecture-04)

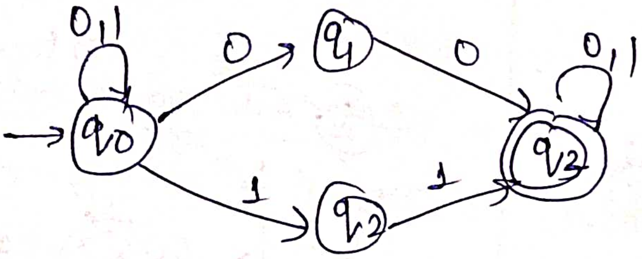
Q. Convert ~~RA~~ RE in FA.

$(0+1)^*(00+11)(0+1)^*$

Ans



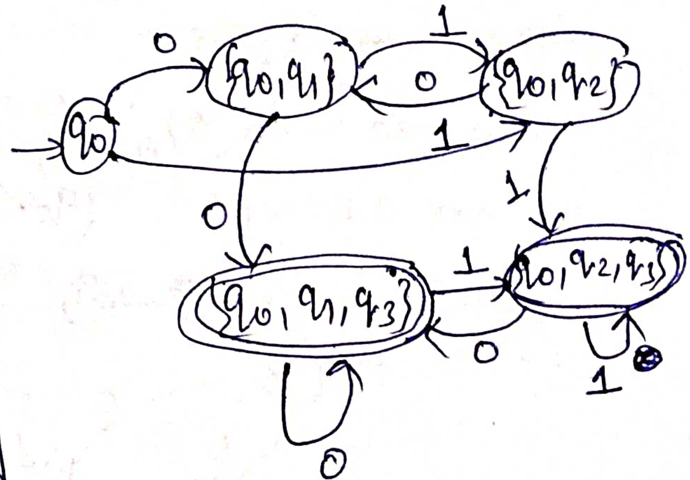




DFA TT

P.S	N.S 0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$\{q_0,q_1\}$	$\{q_0,q_1,q_3\}$	$\{q_0,q_2\}$
$\{q_0,q_2\}$	$\{q_0,q_1\}$	$\{q_0,q_2,q_3\}$
$\{q_0,q_1,q_3\}$	$\{q_0,q_1,q_3\}$	$\{q_0,q_2,q_3\}$
$\{q_0,q_2,q_3\}$	$\{q_0,q_1,q_3\}$	$\{q_0,q_2,q_3\}$

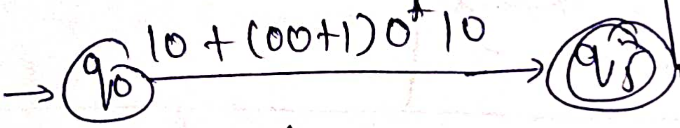
DFA

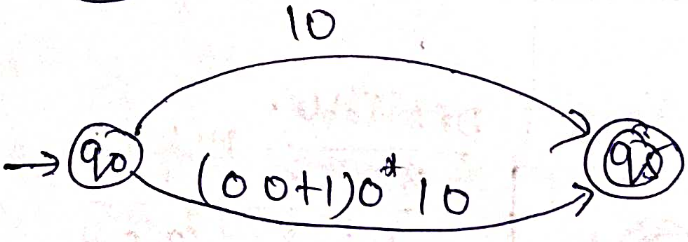


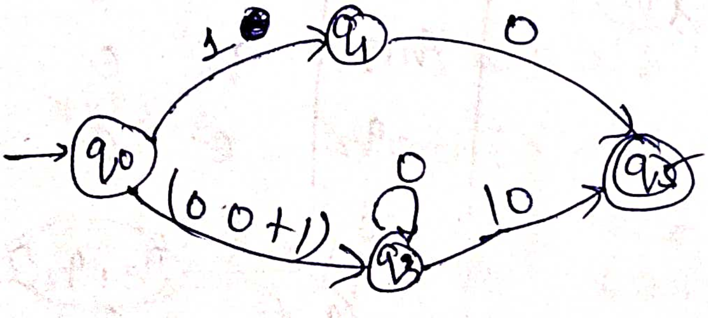
Q. Convert the RE in FA.

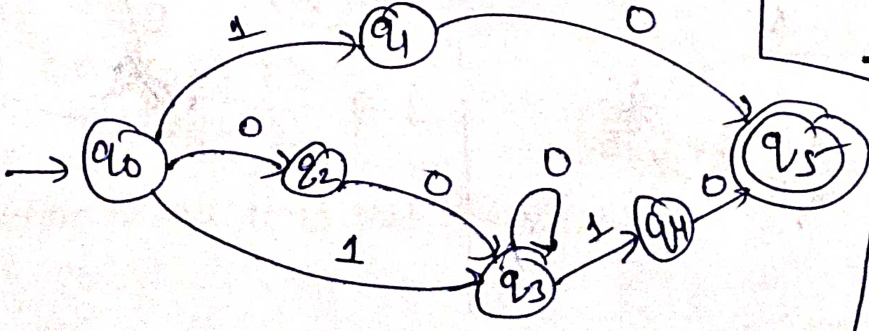
$10+(00+1)0^*10$

Ans



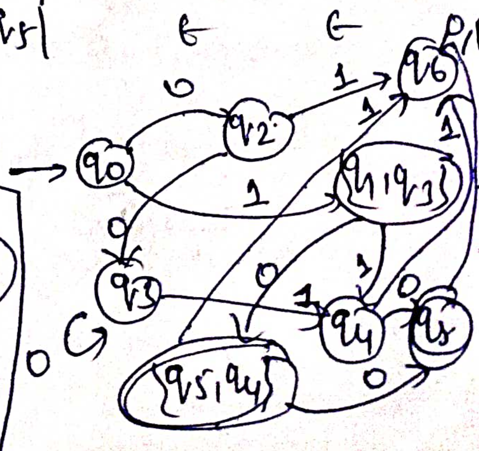






DFA Table

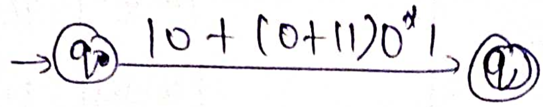
P.S.	N.S 0	1
$\rightarrow q_0$	q_2	$\{q_1,q_3\}$
q_2	q_3	ϵ
$\{q_1,q_3\}$	$\{q_5,q_4\}$	$\{q_4\}$
q_3	q_3	q_4
$\{q_5,q_4\}$	q_5	ϵ
q_4	q_5	ϵ
q_5	ϵ	ϵ

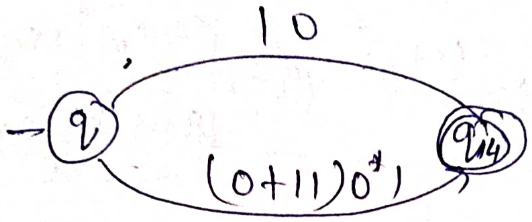


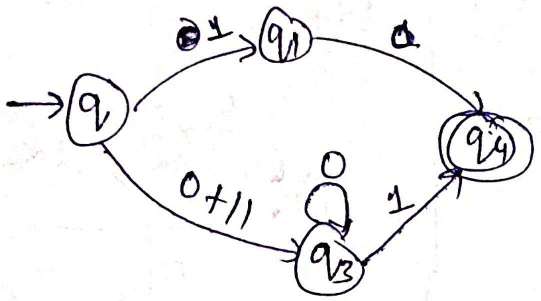
(~~Lecture 01~~)

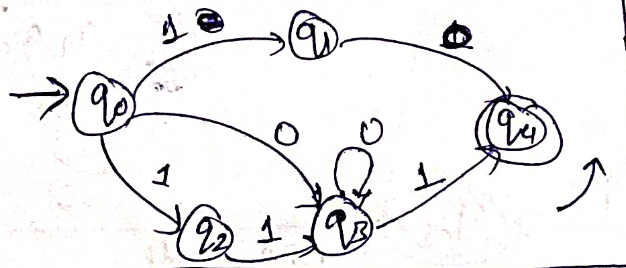
Q. Convert the Regular Exp. into FA.

$10 + (0+11)0^*1$



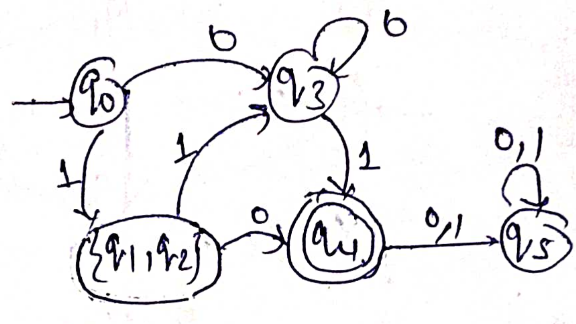






DFA Table:

Q	0	1
	N.S.	
$\rightarrow q_0$	$\{q_3\}$	$\{q_1, q_2\}$
q_3	q_3	q_4
$\{q_1, q_2\}$	$\{q_4\}$	$\{q_3\}$
q_4	$\in$	$\in$

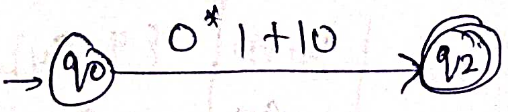


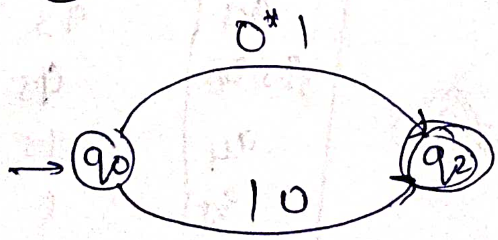
FA for RE

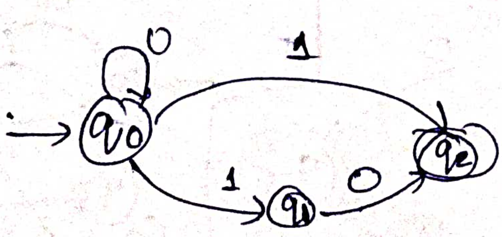
Convert the RE in FA

Q. $0^*1 + 10$

Ans

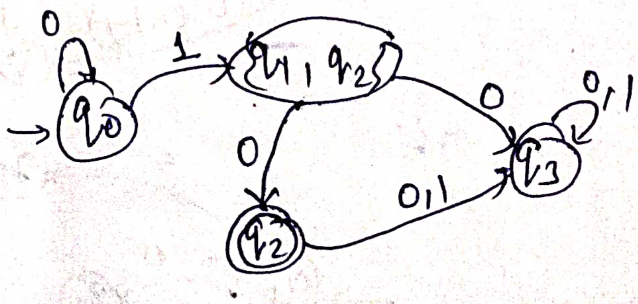






DFA Table:

P.S	0	1
	N.S	
$\rightarrow q_0$	q_0	$\{q_1 q_2\}$
$\{q_1, q_2\}$	q_2	$\in$
q_2	$\in$	$\in$



(Lecture-05)

Pumping Lemma for Regular expression

→ Pumping Lemma is used to prove that a language is NOT REGULAR

→ It can not be used to prove that a language is Regular

$$\text{Language} \rightarrow \text{Pumping Lemma Test} \xrightarrow{\text{Pass}} \text{undecidable (Reg / Not Reg)}$$

$\downarrow$ fail

(Decidable) Not Regular

<u>Pumping Lemma</u>:- If L is an infinite language then there exists some positive integer 'n' (Pumping length) such that any string $w \in L$ has length greater then equal to 'n' i.e $|w| \geq n$ then w can be divided into three parts, $w = xyz$ satify following conditions.

i) for each $i \geq 0$, $xy^iz \in L$

ii) $|y| > 0$ and

iii) $|xy| \leq n$.

<u>Ex</u>:→ $a^n b^{2n}$ $n \geq 0$ no of b's is 2 times of no of a's

<u>Soln</u>:→ Assume that Language is Regular

$w = \underset{x}{aa}\ \underset{y}{bb}\ \underset{z}{bb} \in L$ $p = 2$ (Pumping length) - P = Pumping Length

$w = a^p b^{2p}$

i) $xy^iz \in L$ for each $i \geq 0$

2) $|y| > 0$

3) $|xy| \leq p$ $n = 2$ (P.L)

$\underset{x}{aa}\ \underset{y}{bb}\ \underset{z}{bb}$ $(bb)^i$ $i = 1$ $4 \leq p$ (Not satiesfy)

$(bb)^i$ $i = 2$ (Pump up the value of y)

$\underset{x}{aa}\ \underset{y}{bbbb}\ \underset{z}{bb} \notin L$ (here no of b's is 3 times of no of a's)

Contradiction → so it is Not Regular.

Ex-2 Using pumping Lemma prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is not Regular.

Proof:- Assume that A is Regular

Pumping Length = P

$w = a^P b^P \Rightarrow w = aaaaaaabbbbbbb$

$x \quad y \quad z$

$P = 7$

Case-1 The y is in the a part

$\underbrace{aa}_{x}\underbrace{aaaaa}_{y}\underbrace{bbbbbbb}_{z}$

1) $xy^iz \Rightarrow xy^2z$

aa aaaa aaaa a bbbbbbb

$a' 11 \neq 7' b$

Case-2 The y is in the b part

$\underbrace{aaaaaaabb}_{x}\underbrace{bbbb}_{y}\underbrace{b}_{z}$

$xy^iz = xy^2z$

aaaaaaa bbbb bbbb bbb

$7a \neq 11b$

Case-3 The y is in the a and b part

$\underbrace{aaaaa}_{x}\underbrace{aabb}_{y}\underbrace{bbbbb}_{z}$

$xy^iz \Rightarrow xy^2z$

aaaaa aabb aabb bbbbb

This does not follow the Pattern $a^n b^n$

So $xy^iz \notin L$

2) $|y| > 0$

3) $|xy| \leq P \qquad P = 7$

Case 1 $\quad |xy| = 6 \quad$ (satisfy)

Case 2 $\quad |xy| = 13 \quad$ (Not satisfy)

Case 3 $\quad |xy| = 9 \quad$ (Not satisfy)

Contradiction

So Language A is Not Regular.

Pumping Lemma for Reg. Language :-

Q Prove that $L = \{a^P \mid P \text{ is Prime}\}$ is not Regular.

Soln :- Let L is Regular language.

Pumping Length N

Let $n = 3$

$w = aaa$

Prime No :->
* Divide by itself or only by 1
e.g. 2, 3, 5, 7, 9, 11 --

$L = \{aa, aaa, aaaaa \dots\}$

$w = \underset{x}{a}\,\underset{y}{a}\,\underset{z}{a}$

(i) $xy^iz \quad i \geq 0$

if $i = 0 \quad a \cdot a^0 \cdot a \Rightarrow aa \in L$

if $i = 1 \quad a \cdot a^1 \cdot a = a\underset{y}{a}a \in L$

if $i = 2 \quad a \cdot a^2 \cdot a = \underset{x}{a}\,\underset{y}{aa}\,\underset{z}{a} \notin L$

(ii) $|y| > 0 \quad 2 > 0$ (satisfy)

(iii) $|xy| \leq n \quad 3 \leq 3$ (satisfy)

so $xy^iz \notin L$ this is contradiction so Language is not Regular.

(Lecture-06)

Regular Languages are Decidable

Regular languages are decidable: given any two regular languages A and B, an algorithm can determine whether A and B contain the same strings.

The decision algorithm exploits the fact that *set operations* can be performed on regular languages, based on transformations of finite automata. Because a regular language in any form (regular expression, DFA, and NFA) can be freely converted to any other form, these operations on automata are fully general.

Because these set operations yield a finite automaton as a result, the results of set operations on regular languages are also regular languages.

Let's say that A and B are finite automata recognizing regular languages A and B. (I.e., we use "A" to mean both a regular language and a finite automaton that recognizes language "A".) We can transform the automata A and/or B to construct an automaton that recognizes the languages

$A \cup B$, the union of A and B

A^c, the complement of A, meaning all strings (over some alphabet) that are *not* in A

$A \cap B$, the intersection of A and B

A - B (set difference), all strings that are in A but not in B

In addition to the possibility of performing set operations on finite automata, it is also possible to check any finite automaton to find out whether or not it recognizes a nonempty set of strings. Using this check, we can find out, for any regular language, whether or not that language is empty.

A decision procedure to check the equivalence of regular languages A and B is

$(A = B) \equiv ((A - B = \varnothing) \text{ and } (B - A = \varnothing))$

In other words, A is equivalent to B if and only if the set differences (A - B) and (B - A) are both empty.

Constructions for Set Operations

Here are constructions for set operations on regular languages.

Union (A ∪ B): The basic idea is to create new start and accepting states, connect the new start state to the start states for A and B using epsilon transitions, and connect the accepting states of A and B to the new accepting state using epsilon transitions. After these modifications are done, all of the original start and accepting states of A and B become non-start, non-accepting states. The resulting NFA recognizes the union of A and B.

Complement (A^c): Given a DFA recognizing language A, create an explicit "reject" state, which is a non-accepting state. In all states of A where there is no explicit transition on a particular input symbol, create an explicit transition to the reject state on that symbol. In the reject state, self-transitions on each possible input symbol lead back to the reject state. The result of these transformations is a DFA that recognizes the same language as the original DFA, but every state has a transition on every input symbol. By changing each accepting state to a non-accepting state, and changing each non-accepting state to an accepting state, we create a DFA that recognize the complement of A.

Intersection (A ∩ B): Intersection can be synthesized using the constructions for union and complement:

$A \cap B = ((A^c) \cup (B^c))^c$

In other words, the intersection of A and B is the complement of the union of the complements of A and B.

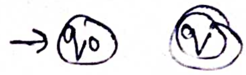
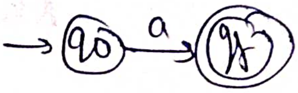
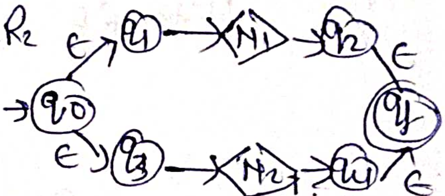
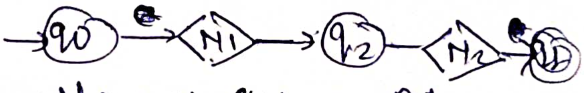
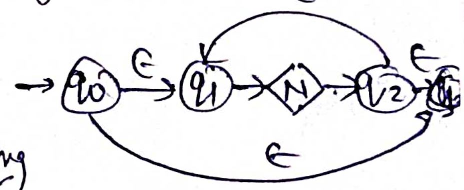
Difference (A - B): Difference can be synthesized using the constructions for intersection and complement

$A - B = A \cap B^c$

In other words, A - B is the set of all strings that are in A and also in the complement of B.

(Lecture-07)

Comparative study of RE, Regular set & finite Automata

	Regular expression	Regular set	finite Automata
(1)	ϕ	{ }	
(2)	ϵ	{ϵ}	
(3)	Every a in Σ is a RE	{a}	
(4)	If r_1 & r_2 are regular expression then (r_1+r_2) is a RE	$R_1 \cup R_2$ {where R_1 & R_2 are Regular set corresponding r_1 & r_2 respectively	 N_1 & N_2 are FA accepting R_1 & R_2 respectively
(5)	If $r_1 \cdot r_2$ is a RE	"	 where N_1 & N_2 are FA accepting R_1 & R_2 resp.
(6)	r^* is a RE	R^* (where R is the Regular set corresponding to r)	 N ~~xxx~~ is FA accepting R.

UNIT II: (Important Questions)

1. State pumping lemma for regular languages. Use pumping lemma to prove that the language L, defined as follows, is not regular. L = {0m 1n | m and n are positive integers and m ≠ n}.
2. Explain the difference between Moore machine and Mealy machine. Describe with the help of an example how a Moore machine can be converted to a Mealy machine.
3. Write regular expression for set of all strings such that number of a's divisibleby 3 over Σ = {a,b}(UPTU 2018-19)
4. Let r1 and r2 be two regular expressions defined as follows :r1 = (00*1)*1 and r2 = 1 + 0(0 + 10)*11. Prove that r1 = r2. (UPTU 2012-13)
5. Prove that the language L = {0n | n is prime} is not regular.
6. Following Grammar generates language of Regular Expression :∈0B|1B|→ B∈0A/→A1B A→0*1 (0+1)* S Give leftmost and rightmost derivation of strings 00101.
7. Prove that following are not Regular Languages : (i) {on | n is perfect square}.
8. The set of strings of form 0i 1j such that the greatest common divisor of i and j is 1.
9. Design a Transducer (Mealy or Moore) Machine to compute multiplication of two n-bit binary numbers.
10. Write the statement of Pumping Lemma and using it prove that the language $\{a^nb^nc^n \mid n>=1\}$ is not regular. (UPTU 2013-14)
11. Findthe regularexpression corresponding tothe finite automatagiven bellow:(UPTU 2018-19)